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A theoretical upper limit to Coble creep strain resulting from concurrent grain growth

B. BURTON
Nuclear Electric plc, Berkeley Technology Centre, Berkeley, Gloucestershire GL13 9PB, UK

The rate of diffusional creep varies with grain size x, either as $1/x^2$ or $1/x^3$, depending on whether lattice or grain boundary diffusion is dominating. Since the rate of grain growth is proportional to $1/x^p$, where $p \geqslant 1$, the creep and grain growth relationships can be combined to predict the transient creep that results from the two processes operating concurrently. An important result is obtained for grain boundary diffusion creep (Coble creep), where two regimes of behaviour are predicted depending on the value of p. For normal grain growth ($p = 1$) and up to a critical value $p = 2$, the transient gives rise to an upper limit to the grain boundary diffusional creep strain. For $p > 2$, no limiting strain is predicted. The role of the limiting strain is discussed in the context of the various experimental attempts that have been made to verify the Coble mechanism.

1. Introduction

Diffusional creep is often the predominant deformation mode in fine-grained polycrystalline materials at low stress levels. When mass transport occurs by lattice diffusion, the creep rate is predicted by the Nabarro–Herring equation [1, 2]:

$$\dot{\varepsilon}_{NH} = B_{NH}\sigma\Omega D/x^2kT \tag{1}$$

where σ is the stress, Ω the atomic volume, D the lattice diffusion coefficient, x the grain diameter, k the Boltzmann constant and T the absolute temperature. B_{NH} is a numerical constant which depends on grain geometry. When mass transport occurs by grain boundary diffusion, the creep rate is predicted by the Coble equation [3]:

$$\dot{\varepsilon}_C = B_C\sigma\Omega\delta D_g/x^3kT \tag{2}$$

where D_g is the grain boundary diffusion coefficient and δ the boundary width. B_C is a numerical constant which again depends on grain geometry. Both mechanisms may be considered to act independently so that the overall creep rate is the sum of Equations 1 and 2, with one or the other providing the dominant contribution depending on grain size and temperature. Since Equation 2 has the stronger grain size dependence and the activation energy for boundary diffusion is less than that for lattice diffusion, Coble creep is predicted to dominate at lower temperatures and/or for fine grain size.

Although it is almost 30 years since the Coble mechanism was proposed, there have been relatively few experimental verifications of it. Those experiments which do confirm the mechanism were performed mainly on highly sensitive specimen configurations, such as helically coiled wires, where the total strains were very small (e.g. [4–6]). One of the experimental difficulties is that concurrent grain growth tends to occur during the creep testing of materials with the fine grain sizes necessary for Coble creep to dominate. The aim of this paper is to evaluate theoretically the influence of concurrent grain growth and to indicate the limitations imposed on the strain achievable by Coble creep.

2. Concurrent grain growth during diffusional creep

During normal grain growth the average grain size x of a material changes at a rate described by the relationship

$$\mathrm{d}x/\mathrm{d}t = FM \tag{3}$$

where $M = D_g/kT$ is the atomic mobility in the grain boundary and F is the driving force, given by

$$F = \frac{\Omega}{\delta}\left(\frac{2\gamma}{x}\right) \tag{4}$$

where γ is the grain boundary energy. Integration of Equation 3 gives the normal grain growth law:

$$x^2 = x_0^2 + 2\left(\frac{2\gamma\Omega}{\delta}\right)\left(\frac{D_g}{kT}\right)t \tag{5}$$

where x_0 is the grain size at $t = 0$. This time dependence of the grain size can be written in a form more convenient for incorporation into the creep equations as follows:

$$x = x_0(1 + \beta t)^{1/2} \tag{6}$$

where $\beta = 4\gamma\Omega D_g/\delta x_0^2kT$.

Consider first the case of Nabarro-Herring creep. From Equations 1 and 6, the creep rate varies with time according to:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{B_{NH}\sigma\Omega D}{x_0^2(1+\beta t)kT} = \frac{\dot{\varepsilon}_0}{1+\beta t} \tag{7}$$

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This equation can be integrated with the conditions that $\varepsilon = 0$ at $t = 0$, to give the dependence of strain upon time as follows:

$$\varepsilon = \frac{\dot{\varepsilon}_0}{\beta}[\ln(1 + \beta t)] \quad (8)$$

Alternatively, the creep rate can be expressed as a function of creep strain by combining Equations 7 and 8. Thus

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp(-\varepsilon\beta/\dot{\varepsilon}_0) \quad (9)$$

The Nabarro–Herring creep strain is thus predicted to vary logarithmically with time according to Equation 8 and the rate to decrease exponentially with creep strain according to Equation 9.

Consider next the case of Coble creep. The time dependence of the creep rate is obtained from Equations 2 and 6 as follows:

$$\frac{d\varepsilon}{dt} = \frac{B_C \sigma \Omega \delta D_g}{x_0^3 (1 + \beta t)^{3/2} kT} = \frac{\dot{\varepsilon}_0}{(1 + \beta t)^{3/2}} \quad (10)$$

The dependence of strain upon time can be obtained by integration as before and this gives

$$\varepsilon = \frac{2\dot{\varepsilon}_0}{\beta}[1 - (1 + \beta t)^{-1/2}] \quad (11)$$

The creep rate can be expressed as a function of the creep strain by combining Equations 10 and 11. Thus

$$\dot{\varepsilon} = \dot{\varepsilon}_0[1 - (\varepsilon\beta/2\dot{\varepsilon}_0)]^3 \quad (12)$$

The behaviour predicted by Equations 11 and 12 for Coble creep is fundamentally different to the case for Nabarro–Herring creep. Equation 11 indicates that as $t \to \infty$ the strain tends to a limiting value given by

$$\varepsilon_{\rm lim} = \frac{2\dot{\varepsilon}_0}{\beta} = \frac{B_C \sigma \delta^2}{2\gamma x_0} \quad (13)$$

Substitution of $\varepsilon = \varepsilon_{\rm lim}$ in Equation 12 confirms that the creep rate is zero at this limiting value. Noting that the expression for one elastic deflection is $\varepsilon_{\rm el} = \sigma/G$, where G is the shear modulus, and that $\delta \simeq 2b$, where b is the atomic size, this enables $\varepsilon_{\rm lim}$ to be expressed as a function of an elastic strain. Also taking $B_C = 16$ [7], this gives

$$\varepsilon_{\rm lim}/\varepsilon_{\rm el} = 32Gb^2/\gamma x_0 \quad (14)$$

By using the values $\gamma = 0.5\ \mathrm{N\,m^{-1}}$, $G = 10^{11}\ \mathrm{N\,m^{-2}}$ and $b = 3 \times 10^{-10}$ m, the limiting strain can be evaluated as a function of x_0. For a typical grain size of 10 μm the limiting strain is less than 10% of an elastic deflection.

The transient behaviour predicted above is shown graphically in Fig. 1 for a typical f.c.c. metal with an initial grain size $x_0 = 10 \times 10^{-6}$ m at the homologous temperature $T/T_m = 0.75$. Other values used in calculating the curves were as follows: $D = 5.4 \times 10^{-5} \exp(-18.4 T_m/T)\ \mathrm{m^2\,s^{-1}}$ [8], $\delta D_g = 9.4 \times 10^{-15} \times \exp(-10 T_m/T)\ \mathrm{m^3\,s^{-1}}$ [8], $k = 1.38 \times 10^{-23}\ \mathrm{N\,m\,K^{-1}}$, $T_m = 1356$ K, $B_{NH} = 10$ and $\sigma = 10\ \mathrm{MN\,m^{-2}}$. The strain is expressed as a fraction of an elastic deflection and the time is plotted in the dimensionless form $\dot{\varepsilon}_{\rm init} t/\varepsilon_{\rm el}$, where $\dot{\varepsilon}_{\rm init}$ is the initial creep rate (the sum of the Nabarro–Herring and Coble contributions). It is clear that Coble creep dominates initially but decays more rapidly than Nabarro–Herring creep. The Coble contribution eventually saturates at a strain of about 0.06 of an elastic deflection. It is clear by inspection of Equations 1 and 2, that Nabarro–Herring creep rate dominates over the Coble rate when the grain size exceeds the critical value $x = x_{\rm crit} = B_C \delta D_g / B_{NH} D$. From Equation 6, this occurs at a time $[(x_{\rm crit}/x_0)^2 - 1]/\beta$.

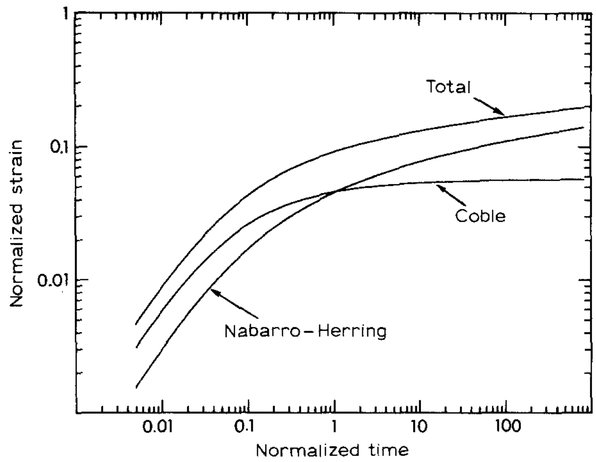

Figure 1 Variation of strain with time for a material undergoing concurrent grain growth. The strain is plotted in the normalized form $\varepsilon/\varepsilon_{\rm el}$ where $\varepsilon_{\rm el}$ is an elastic deflection and the time is plotted in the dimensionless form $\dot{\varepsilon}_{\rm init} t/\varepsilon_{\rm el}$, where $\dot{\varepsilon}_{\rm init}$ is the initial creep rate.

3. Influence of a non-linear grain growth law

The above analysis is for the normal case where the rate of grain growth depends linearly on the product of the force and the mobility, when dx/dt is proportional to $1/x$. Under other circumstances, particularly when grain growth is impeded by the presence of boundary inclusions such as particles or voids, it can be more sensitive to grain size. A grain growth expression of the following type is then often appropriate:

$$dx/dt = A/x^p \quad (15)$$

where the exponent $p > 1$ and A is a rate constant. The integrated form of this equation is given by

$$x = x_0(1 + \alpha t)^{1/(p+1)} \quad (16)$$

where $\alpha = (p + 1)A/x_0^{p+1}$. Equation 16 can be substituted into the creep rate Equations 1 and 2 as before and the resulting expressions can be integrated.

For Nabarro–Herring creep, the dependence of strain upon time may then be derived as follows:

$$\varepsilon = \frac{\dot{\varepsilon}_0(p+1)}{\alpha(p-1)}[(1 + \alpha t)^{(p-1)/(p+1)} - 1] \quad (17)$$

and the dependence of creep rate upon creep strain is given by

$$\dot{\varepsilon} = \dot{\varepsilon}_0\left[1 + \frac{\varepsilon\alpha(p-1)}{\dot{\varepsilon}_0(p+1)}\right]^{-2/(p-1)} \quad (18)$$

The value of $(p-1)/(p+1)$ in Equation 17 is always positive for all values of p greater than unity. Thus, similar to the case of normal grain growth, the Nabarro–Herring creep strain is predicted never to reach a limiting value.

For Coble creep, a corresponding derivation leads to equivalent expressions. Firstly for the strain–time dependence

$$\varepsilon = \frac{\dot{\varepsilon}_0(p+1)}{\alpha(p-2)}\left[(1+\alpha t)^{(p-2)/(p+1)} - 1\right] \quad (19)$$

and secondly for the dependence of creep rate upon creep strain

$$\dot{\varepsilon} = \dot{\varepsilon}_0\left[1 + \frac{\varepsilon\alpha(p-2)}{\dot{\varepsilon}(p+1)}\right]^{-3/(p-2)} \quad (20)$$

In this case, two regimes of behaviour are revealed, depending on the value of p. When $p < 2$, the value of $(p-2)/(p+1)$ is negative and a limiting Coble creep strain is predicted as for the case of normal grain growth. It is given by

$$\varepsilon_{\lim} = \dot{\varepsilon}_0(p+1)/\alpha(2-p) \quad (21)$$

When $p > 2$, the value of $(p-2)/(p+1)$ is positive and no limiting strain is then predicted.

Note that for the special case when $p = 2$, integration of the creep rate–time expression gives a logarithmic strain–time dependence and an exponential decay of creep rate with creep strain as follows:

$$\varepsilon = \frac{\dot{\varepsilon}_0 x_0^3}{3A}\ln(1 + 3At/x_0^3) \quad (22)$$

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp(-3A\varepsilon/\dot{\varepsilon}_0 x_0^3) \quad (23)$$

4. Discussion

The novel feature revealed by the present analysis is the upper limit to Coble creep strain that is predicted for grain growth exponents in the range $1 \leqslant p < 2$. The limiting strain can be small. For the numerical example given, it is less than one elastic deflection. In conventional uni-axial creep tests, the sensitivity of the extensometers may be insufficient to measure the strains with the accuracy required. There are two choices then available to the experimentalist intent on confirming the existence of Coble creep. The first is to use a more sensitive specimen configuration such as a helical spring. This enables tests to be performed relatively easily at small strains so that sufficient data points can be collected to determine the creep rate over a time scale in which grain growth is negligible. The other choice is to ensure that grain growth is impeded, for example by the presence of boundary particles or voids. The presence of such inclusions can lead to further difficulties however. Particles are known to inhibit diffusional creep as well as grain growth [9] so that their addition to stabilize the grains may totally inhibit diffusional creep in certain cases. In other cases, there may exist only a narrow range over which experiments can be performed, where there are sufficient particles present to stabilize the grain size but insufficient to inhibit creep. When voids or bubbles are present on grain boundaries, such as sintering cavities in ceramic materials or fission gas bubbles in nuclear materials, these may suppress grain growth, but on the other hand, the diffusional growth or sintering of them may give rise to a creep strain of the same order as the Coble strain [10]. In such a case, it is then necessary to separate the two contributions to creep strain, for example by using density measurements.

The present calculations emphasize the importance of taking account of concurrent grain growth in assessments of the creep behaviour of materials at low stress levels. This is particularly so in the case of Coble creep in materials where normal grain growth occurs. An upper limiting Coble strain might then exist. Since no limiting strain is predicted for Nabarro–Herring creep and the overall diffusional creep strain is equal to the sum of the grain boundary and lattice contributions, it is clear that Nabarro–Herring creep will always dominate eventually.

5. Conclusions

1. The transient diffusional creep that results from concurrent grain growth can be predicted by substituting the grain growth expression into the diffusional creep equations.
2. For grain boundary diffusional creep (Coble creep), the type of transient behaviour depends on the value of the exponent in the grain growth equation $dx/dt \propto 1/x^p$. For $1 \leqslant p < 2$ a theoretical upper limit to Coble creep strain exists, whose value can be less than an elastic deflection. When $p \geqslant 2$, no upper limit exists to the strain achievable.
3. The existence of the limiting strain may impose a severe restraint on the possibility of detecting Coble creep by using conventional uni-axial creep testing.
4. It may not be possible to extend the range of dominance of Coble creep by adding particles to stabilize the grain size, since these may cause a total inhibition of diffusional creep.
5. In materials whose grain growth is impeded by voids or bubbles, the diffusional size-change of these may give rise to a strain contribution of the same order as that due to Coble creep.
6. For lattice diffusional creep (Nabarro–Herring creep), no upper limit to the creep strain is predicted. Concurrent grain growth leads to a transient state where the rate decreases steadily with time.

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